

Report: Equations in Physics

Kenn K. Q. Zhang

Abstract

For numerical purposes, some equations in macroscopic physics and engineering, atomic and particle physics, and cosmic physics are classified based on temporal and spatial derivatives.

1 Zero Derivative in Time

1.1 Navier Equation in Elastostatics

$$\nabla \cdot \tilde{\sigma} + \rho \vec{f} = 0$$

1.2 Dirac Equation in Quantum Electrodynamics

Dirac equation for spin $\frac{1}{2}$ objects is a first derivative in time and first derivative in space equation, but with 4-vector it is considered as a static equation – from numerical standpoint

$$i\hbar\gamma^\mu\partial_\mu\psi - mc\psi = 0$$

1.3 Einstein Field Equation in General Relativity

Einstein gravitational field equation for spin 2 objects is a zero derivative equation in spacetime

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^2}T_{\mu\nu}$$

2 First Derivative in Time

2.1 Navier-Stokes Equation for Compressible Flows

$$\begin{aligned} \rho \frac{\partial u_j}{\partial t} + \rho u_k \frac{\partial u_j}{\partial x_k} &= -\frac{\partial p}{\partial x_j} + \frac{\partial}{\partial x_j} \left(\lambda \frac{\partial u_k}{\partial x_k} \right) \\ &+ \frac{\partial}{\partial x_i} \left[\mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right] + \rho f_j \end{aligned}$$

2.2 Schrodinger Equation in Quantum Mechanics

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi + V\psi$$

3 Second Derivative in Time

3.1 Lighthill Equation in Aeroacoustics

Lighthill scalar wave equation for aeroacoustics reads

$$\frac{\partial^2 \rho}{\partial t^2} - c_0^2 \nabla^2 \rho = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j}$$

3.2 Elastodynamics (Seismic Wave)

The vector wave equation in solids reads

$$\rho \frac{\partial^2 \vec{u}}{\partial t^2} = \mu \nabla^2 \vec{u} + (\mu + \lambda) \nabla (\nabla \cdot \vec{u}) + \rho \vec{f}$$

3.3 Klein-Gordon Equation in Quantum Electrodynamics

The complex variable wave equation for spin 0 objects reads

$$-\frac{1}{c^2} \frac{\partial^2 \psi}{\partial t^2} + \nabla^2 \psi = \left(\frac{mc}{\hbar}\right)^2 \psi$$

3.4 Maxwell Equations in Electrodynamics

The coupled wave equation for electromagnetism reads

$$\begin{aligned} \nabla^2 \phi - \frac{1}{c^2} \frac{\partial^2 \phi}{\partial t^2} &= -\rho / \epsilon_0 \\ \nabla^2 \vec{A} - \frac{1}{c^2} \frac{\partial^2 \vec{A}}{\partial t^2} &= -\mu_0 \vec{J} \end{aligned}$$