# A Chebyshev Collocation Spectral Method for Glow Discharge Plasma Physics

Kenn K. Q. Zhang

## 1 Introduction

The glow discharge in two parallel plates (Figure 1) is a popular problem in plasma physics research. The neutral medium between two plates is, all the sudden, subject to an applied strong electric field. Consequently, a fraction of electrons move toward the anode while a fraction of ions move toward the cathode. Although electrons are absorbed by the anode quite peacefully, ions are not. In the vicinity of the cathode, ions have gained much momentum so that they impinge the cathode and cause the second emission of electrons and ions. Also, all ions with large momentum collide the neutral medium and produce electrons and ions. Therefore, a complex physical system is formed with high concentration of electrons, and ions in particular, in the sheath layer of cathode. In stead of the dispersed approach of Monte Carlo which might be an option, here the continuous approach is adopted and the whole physical process is modelled by the diffusion-drift theory. In the next section the complete set of mathematical equations [1] are listed, followed by a section on computational aspects.

## 2 Mathematical Models

Mathematical equations for the electric potential $\phi$, electron density $n^-$, and ion density $n^+$ are described in the following three subsections, where $x$, $y$, and $t$ are independent variables and the remaining unspecified quantities are either parameters or constants [1].

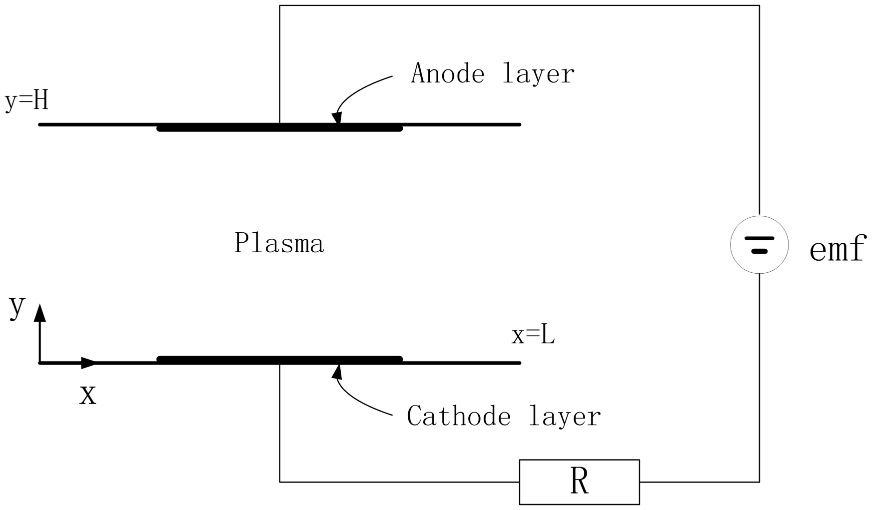


Figure 1: Configuration for the glow discharge between two parallel plates.

### 2.1 Equations for electric potential

For the electric potential $\phi$

$$\nabla^2 \phi = \epsilon(n^- - n^+), \tag{1}$$

with BCs

$$\frac{\partial \phi}{\partial x} = 0, \quad on\ x = 0, L, \tag{2}$$

$$\phi = 0, \quad on\ y = 0, \tag{3}$$

$$\phi = emf - R \int_0^L e\ \mu^- n^- |\nabla \phi|\ dx, \quad on\ y = H, \tag{4}$$

### 2.2 Equations for electron density

For the electron density $n^-$

$$\frac{\partial n^-}{\partial t} + \nabla \cdot \vec{\Gamma}^- = A\ p\ e^{-\frac{Bp}{|\nabla \phi|}} \left| \vec{\Gamma}^- \right| - \beta n^- n^+, \tag{5}$$

where $\Gamma^-$ is defined as

$$\Gamma_x^- = \mu^- \frac{\partial \phi}{\partial x} n^- - D^- \frac{\partial n^-}{\partial x}, \tag{6}$$

$$\Gamma_y^- = \mu^- \frac{\partial \phi}{\partial y} n^- - D^- \frac{\partial n^-}{\partial y} \tag{7}$$

The corresponding boundary conditions for the electron density are

$$\frac{\partial n^-}{\partial x} = 0, \quad on\ x = 0, L, \tag{8}$$

$$n^- = \gamma \frac{\mu^+}{\mu^-} n^+, \quad on\ y = 0, \tag{9}$$

$$\frac{\partial n^-}{\partial y} = \frac{1}{D^-}\left(\mu^- \frac{\partial \phi}{\partial y} - \frac{V^-}{2}\right) n^-, \quad on\ y = H, \tag{10}$$

### 2.3 Equations for ion density

For the ion density $n^+$

$$\frac{\partial n^+}{\partial t} + \nabla \cdot \vec{\Gamma}^+ = A\ p\ e^{-\frac{Bp}{|\nabla \phi|}} \left|\vec{\Gamma}^-\right| - \beta n^- n^+, \tag{11}$$

where $\Gamma^+$ is defined as

$$\Gamma_x^+ = -\mu^+ \frac{\partial \phi}{\partial x} n^+ - D^+ \frac{\partial n^+}{\partial x}, \tag{12}$$

$$\Gamma_y^+ = -\mu^+ \frac{\partial \phi}{\partial y} n^+ - D^+ \frac{\partial n^+}{\partial y} \tag{13}$$

The corresponding boundary conditions for the ion density are

$$\frac{\partial n^+}{\partial x} = 0, \quad on\ x = 0, L, \tag{14}$$

$$\frac{\partial n^-}{\partial y} = 0, \quad on\ y = 0, \tag{15}$$

$$n^+ = 0, \quad on\ y = H, \tag{16}$$

## 3 Computational Challenges

The system described in the previous section is computationally highly sensitive, in other words, it is an ill-natured system. The reasons are as follows.

1. Two different time scales for electrons and ions demands a computationally efficient method. In other words, this is an excellent example of multi scale problems.

2. The very high gradient in the cathode layer demands a very accurate method capable of dealing with shocks.

3. The integral boundary condition equation (4), albeit a well-natured Fredholm second kind, actually is a nuisance. This may be a consequence of strong coupling among the electric potential, electron density, and ion density.

4. The interplay between the large advection terms and the highly nonlinear source terms demands a very accurate method. Contrary to many familiar systems in fluid mechanics where an inaccuracy of a method usually yet gives qualitatively correct result, here the inaccuracy can turn the system into completely nonphysical, as encountered in many computation of chemically reactive flows.

The Chebyshev collocation spectral method, where collocation points clustered near boundaries, is tentatively picked on to tackle this glow discharge problem. In the future, a discontinuous spectral element method might be used to solve this problem and other sensitive systems, even those involving complex moving boundaries.

## References

[1] S. T. Surzhikov and J. S. Shang. Two-component plasma model for two-dimentional glow discharge in magnetic field. *J. Comput. Phys.*, 2004.